

Thermal Decays in a Hot Fermi Gas

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Abstract

We present a study of the decay of metastable states of a scalar field via thermal activation, in the presence of a finite density of fermions. The process we consider is the nucleation of “*droplets*” of true vacuum inside the false one. We analyze a one-dimensional system of interacting bosons and fermions, considering the latter at finite temperature and with a given chemical potential. As a consequence of a non-equilibrium formalism previously developed, we obtain time-dependent decay rates.

I. INTRODUCTION

The dynamics of metastable systems is a subject that may be found in all realms of Physics. In Condensed Matter Physics, in polymer transitions [1] and liquid mixtures [2,3]; in High Energy Physics, in the problem of baryon number violation [4] and in the formation of chiral condensates in heavy ions collisions [5] ; in Astrophysics, in “nucleating planets”; in Cosmology, in the well known Inflationary Models [6]. All these examples have at least one thing in common: the occurrence of metastability, which requires non-equilibrium methods.

As a model for learning something about the qualitative features of the decays of metastable states, we will study a one-dimensional system of interacting fermions and bosons

that starts in a metastable vacuum and gradually decays to the true one. The process considered here will be the nucleation of “bubbles” of true vacuum inside the false one via thermal activation. Our main purpose will be the analysis of the stability of these “bubbles” and the calculation of the decay rate as a function of time, in the presence of a finite density of fermions at finite temperature. As already shown in reference [9], the decay rate is time-dependent as opposed to the time-independent results previously found in the literature [2,7]. This is a direct consequence of the use of the non-equilibrium formalism. The results presented here appear to be in qualitative agreement with some experimental results.

The introduction of fermions that interact with the scalar field has two remarkable features. The first, already used in reference [1], is the preservation of the functional form of the bosonic static solution of the equation of motion for the case with no fermions. Through the inverse scattering methods used here, this fact is rigorously demonstrated. The second is the appearance of metastability within metastability. Besides the metastability which is inherent to our choice of the metastable minimum of the effective bosonic potential, we find that the extrema of such a potential may be metastable “bubbles”. Thus, throughout this article, the metastable state (for example, the first excited state of a *cis*-polymer) may develop defects which correspond to either unstable or metastable “bubbles”.

The paper is organized as follows: in section II, we briefly review the results for just a scalar field, which already appeared in the literature; in section III, we include fermion fields and analyze their influence on the effective theory for the bosons; in section IV, we add the effects of finite temperature and of a chemical potential for the fermions; in section V, we present some comments about the method and our conclusions. Some mathematical definitions are left for a final Appendix.

II. THE BOSONIC CASE: A SUMMARY

In this section we summarize results concerning the decay of metastable bosonic systems. Most of the material presented here may be found in various recent papers [8,9].

A. The Lagrangian

Our bosonic system is represented by a real one-dimensional scalar field, $\phi(x)$. The dynamics is given by a Lagrangian with the following form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \lambda(\phi - \phi_-)^2\phi(\phi - \phi^*) \quad (2.1)$$

where

$$\phi^* \equiv \phi_- \left(1 - \frac{m^2}{2\lambda\phi_-^2}\right) \quad (2.2)$$

The Lagrangian (2.1) has an asymmetric double well potential, $V(\phi) = \lambda(\phi - \phi_-)^2\phi(\phi - \phi^*)$, with $V''(\phi) = m^2$, where m^2 is the squared mass of small oscillations around the local minimum ϕ_- (see Figure 1) and λ is a coupling constant.

B. The “sphaleron” solution

The equation of motion associated with (2.1) has a static solution known as “sphaleron” [10]. It corresponds to a field configuration that starts at the false vacuum ϕ_- , almost reaches the true vacuum ϕ_+ , and returns to ϕ_- . It looks like a “droplet” and its explicit form is given by [9,11]

$$\phi_{sph}(x) = \phi_- + \frac{m}{2\sqrt{2\lambda}} \left\{ \tanh \left[\frac{m}{2}(x - x_{c.m.}) + s_0 \right] - \tanh \left[\frac{m}{2}(x - x_{c.m.}) - s_0 \right] \right\} \quad (2.3)$$

where

$$s_0 \equiv \frac{1}{2} \cosh^{-1} \left(\frac{\varepsilon + 1}{\varepsilon - 1} \right) \quad (2.4)$$

$$\varepsilon \equiv \frac{m^2}{2\lambda\phi_-^2} \quad (2.5)$$

The parameter $x_{c.m.}$ reflects the translational invariance of the equation of motion, ε gives a measure of the difference in depth between the true and the false vacua and s_0 is related to what we may call the radius of the “sphaleron” (see Figure 2). As we will see later, this latter parameter assumes great importance in the analysis of stability.

It proves convenient to expand the field $\phi(x)$ and its conjugated momentum $\pi(x)$ around the “sphaleron” configuration, so as to express the Hamiltonian in this particular basis. Collecting terms up to second order, the Hamiltonian acquires the following form

$$\hat{\mathcal{H}} = E_{sph} + \frac{\hat{\pi}_0^2}{2} + \frac{\hat{\pi}_{-1}^2}{2} - \frac{\Omega^2 \hat{\phi}_{-1}^2}{2} + \frac{1}{2} \sum_{l \geq 1} (\hat{\pi}_l^2 + \omega_l^2 \hat{\phi}_l^2) \quad (2.6)$$

where $\{\omega_l\}$ is the set of eigenvalues of the fluctuation operator. This form of the Hamiltonian clearly isolates the contributions coming from the energy of the “sphaleron” itself, the collection of harmonic oscillators, the translational mode and, most important, the presence of an inverted oscillator with frequency Ω . This last contribution signals the existence of an unstable direction in functional space (associated with ϕ_{-1} , which itself is associated with s_0 [9]) ready to guide the decay. Indeed, a short glance at the behavior of the energy of the “bubble”¹ as a function of its “radius”, s , shows us three possible regimes: for $s < s_0$, the “bubble” shrinks and disappears; for $s > s_0$, the “bubble” grows without limit and, for $s = s_0$, we have the critical “bubble”, ready to fall (see Figure 3).

C. Time evolution and decay rate

As we have seen in the last section, the decay of our metastable vacuum may take place through the nucleation, via thermal activation, of “bubbles” with radius larger than the critical s_0 . In order to obtain the decay rate for this process, we must calculate the probability current along the unstable direction, at the saddle point.

Assuming as our initial density matrix one that represents a set of harmonic oscillators of frequency ω_k centered at the metastable minimum ϕ_- , and integrating over all the degrees of freedom but $l = -1$, one obtains, after some algebraic manipulations, the reduced initial density matrix [9] (projected onto the unstable direction, $l = -1$, of functional space

¹We define here a new structure, that we call “bubble”, that generalizes the “sphaleron” in the sense that the parameter s_0 is promoted to the status of a dynamical variable called s .

$$\rho_r(\eta_{-1}, \eta'_{-1}) = \mathcal{N} \exp \left\{ -\frac{1}{2\hbar} \left[\alpha(\eta_{-1}^2 + \eta_{-1}'^2) + 2\gamma\eta_{-1}\eta_{-1}' \right] \right\} \quad (2.7)$$

where

$$\frac{\alpha}{\hbar} \equiv (K_1)_{-1,-1} - \frac{1}{2} \vec{Q}^T \tilde{K}^{-1} \vec{Q} \quad (2.8)$$

$$\frac{\gamma}{\hbar} \equiv -(K_2)_{-1,-1} - \frac{1}{2} \vec{Q}^T \tilde{K}^{-1} \vec{Q} \quad (2.9)$$

$$\mathcal{N} \equiv \sqrt{E_{sph}} \left[\frac{\det(K/\pi)}{\det(\tilde{K}/\pi)} \right]^{1/2} \quad (2.10)$$

and $\eta_l \equiv \phi_l - \bar{\phi}_l$ ($\phi_- - \phi_{sph}(x - x_0) = \sum_l f_l(x - x_0) \bar{\phi}_l$; $\bar{\phi}_0 = 0$, see [9]). The kernels are defined in the Appendix.

The time evolution of this matrix is dictated by the reduced Hamiltonian

$$\hat{\mathcal{H}}_{-1} = \frac{1}{2} \left[-\hbar^2 \frac{\delta^2}{\delta \phi_{-1}^2} - \Omega^2 \phi_{-1}^2 \right] \quad (2.11)$$

through the Liouville equation. Using the most general gaussian *ansatz* compatible with unitarity for the time-evolved density matrix, one obtains for the decay rate per unit volume (see [9])

$$\begin{aligned} \frac{\Gamma(t)}{L} &= J[saddle] = J[\eta_{-1}]_{\eta_{-1}=0} = \\ &= -\Omega \bar{\phi}_{-1}(0) A(t) \sqrt{E_{sph}} \mathcal{N}(0) \exp \left\{ -\frac{1}{\hbar} \bar{\phi}_{-1}^2(0) [\mathcal{R}(\alpha(0)) + \gamma(0)] B(t) \right\} \end{aligned} \quad (2.12)$$

where

$$A(t) \equiv \frac{W^2}{\Omega^2} \frac{\sinh(\Omega t)}{[\cosh^2(\Omega t) + \frac{W^2}{\Omega^2} \sinh^2(\Omega t)]^{3/2}} \quad (2.13)$$

$$B(t) \equiv \frac{1}{1 + \frac{W^2}{\Omega^2} \tanh^2(\Omega t)} \quad (2.14)$$

The form of the decay rate as a function of time is given in Figure 4. To understand this plot, we should remember our initial hypothesis. We have defined a particular initial state for our system and decided to observe its “natural” evolution, i.e., an out-of-equilibrium

evolution which does not replenish the false vacuum. Therefore, the insignificant overlap of the initial state with the saddle point forces $\Gamma(t)$ to vanish at $t = 0$. On the other hand, for $t > 0$ we have the decay to the true vacuum. Thus, $\Gamma(t)$ must vanish also for $t \rightarrow \infty$. Further discussions of these results can be found in the literature [1,8,9].

III. INCLUDING FERMIONS

A. Effective action

The motivation for the inclusion of fermions in our scheme went beyond a simple generalization of the formalism. It was based on the belief that they would introduce qualitatively different results for the stability of the “bubble” structures. This hope relied on the Pauli exclusion principle. As a consequence of the repulsive interaction of the fermions, we expected to find non-trivial metastable minima in the function $E = E[s]$.

Throughout this section, we will be interested on the effects of fermions on the bosonic field. The natural way to proceed is to obtain, from an original fermion-boson Lagrangian, an effective theory for the bosons. The original Lagrangian is defined as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - [V(\phi) - V(\phi_2)] + \bar{\psi}_a(i \not{\partial} - \mu - g\phi) \psi_a \quad (3.1)$$

where μ is the bare mass of the fermions, g is the coupling constant, $\psi_a(x)$ is the fermion field, a denotes fermion species and ϕ_2 is a local minimum of the potential ²

$$V(\phi) = \frac{g^2}{2}(\phi - \phi_0)^2 \left(\phi + \phi_0 + \frac{2\mu}{g} \right)^2 + j\phi \quad (3.2)$$

where ϕ_0 is a constant and j is an external current, responsible for the asymmetry of the potential even in the purely bosonic case.

We may now integrate the generating functional

²We may find a physical realization of this form of potential in the description of conducting polymers [12].

$$Z = \int [D\phi][D\psi_a][D\bar{\psi}_a] \exp \left\{ i \int_0^T dt \int_{-\infty}^{\infty} dx \mathcal{L}[\phi, \psi_a, \bar{\psi}_a] \right\} \quad (3.3)$$

over the fermions, in order to obtain an effective action for the bosons. Following the methods of *Dashen et al.* [13], we arrive at the following expression for the effective action

$$S_{eff}[\phi] = \int_0^T dt \int_{-\infty}^{\infty} dx \left[\frac{1}{2}(\partial_\mu \phi)^2 - [V(\phi) - V(\phi_2)] \right] + N \sum_i \alpha_i[\phi] - \sum_i n_i \alpha_i[\phi] \quad (3.4)$$

where N represent the number of fermion “species”, n_i the occupation number of the discrete (bound states) fermionic levels and α_i are the Floquet indices, defined such that

$$\psi(x, t + T) = e^{-i\alpha_i} \psi(x, t) \quad (3.5)$$

Assuming the existence of only two symmetric bound states and the possibility of “doping”, which will characterize particular configurations defined by the occupation numbers $n_{\pm 1}$ of the bound states, we may write the effective action as

$$\begin{aligned} S_{eff}[\phi] = & \int_0^T dt \int_{-\infty}^{\infty} dx \left[\frac{1}{2}(\partial_\mu \phi)^2 - [V(\phi) - V(\phi_2)] \right] + \\ & + - \int_0^T dt \int_{-\infty}^{\infty} dx \left[\frac{g^2}{2}(\phi^2 - \phi_2^2) + b^* g^2(\phi - \phi_2) \right] + \\ & + N \sum_{i < -1} (\alpha_i[\phi] - \alpha_i[\phi_2]) - (n_+ - n_-)(\alpha_1[\phi] - \alpha_1[\phi_2]) \end{aligned} \quad (3.6)$$

where we have already introduced counterterms and renormalization conditions [14] (that define b^* through $Z_1(\phi_2, \Lambda, g, \mu, j) = b^* g^2$). Note that n_{\pm} represents the occupation number of particle states only, as opposed to $n_{\pm 1}$ which represents particle and anti-particle states ($n_+ \equiv n_{+1}; n_- \equiv N - n_{-1}$).

B. SPA approximation

In order to obtain time-independent solutions, analogous to the “sphaleron”, it proves useful to rewrite the action (3.6) in terms of scattering data. Using well known results from inverse scattering methods [14–16], we may write the effective action as

$$\begin{aligned} \frac{S_{eff}[\phi]}{T} = & -\frac{I_3}{g^2} - 2 \left[(\phi_2^2 - \phi_0^2) + \frac{2\mu}{g}(\phi_2 - \phi_0) \right] I_1 - \frac{j}{g} I_0 - \\ & - (I_1 - \mu I_0) - b^* g I_0 - \frac{N I_2}{\pi} - (n_+ - n_-)[\omega_0(\phi) - \omega_0(\phi_2)] \end{aligned} \quad (3.7)$$

where $\omega_i \equiv \frac{\phi_i}{T}$ and I_0 , I_1 , I_2 and I_3 have their explicit forms shown in the Appendix. Now, we may extremize the action with respect to the scattering data, namely, the reflection coefficients $r_{\pm}(k)$ and $K_0^{\pm} = (m_F^2 - \omega_0^2)^{1/2}$,

$$\frac{\delta S_{eff}}{\delta r_+} = \frac{\delta S_{eff}}{\delta r_-} = 0 \quad (3.8)$$

$$\frac{\delta S_{eff}}{\delta K_0^+} = \frac{\delta S_{eff}}{\delta K_0^-} = 0 \quad (3.9)$$

From (3.8) we obtain $r_+(k) = r_-(k) = 0$, ie, the potential is reflectionless. Using this, we may write the energy as

$$\begin{aligned} E = m_F \left\{ \frac{8}{3} \left(\frac{m_F}{g} \right)^2 \sin^3 \theta_0 + 2 \left(\frac{N}{\pi} - \frac{\Delta}{m_F} \right) \sin \theta_0 + N \left(1 - \frac{2\theta_0}{\pi} + \frac{\Delta n}{N} \right) \cos \theta_0 - \right. \\ \left. - \left(\frac{\Delta}{m_F} \right) \ln \left(\frac{1 - \sin \theta_0}{1 + \sin \theta_0} \right) - (N + \Delta n) \right\} \end{aligned} \quad (3.10)$$

and the equation for the extrema (the gap equation) as

$$\theta_0 + \frac{\pi}{N} \left(\frac{\Delta}{m_F} \right) \tan \theta_0 + \frac{2\pi}{N} \left(\frac{m_F}{g} \right)^2 \sin 2\theta_0 = \frac{\pi}{2} \left(1 + \frac{\Delta n}{N} \right) \quad (3.11)$$

where we have introduced the following definitions

$$m_F \equiv \omega(\phi_2) = \mu + g\phi_2 = (K_0^2 + \omega_0^2)^{1/2} \quad (3.12)$$

$$\Delta n \equiv n_+ - n_- \quad (3.13)$$

$$\omega_0 \equiv m_F \cos \theta_0 \quad (3.14)$$

$$\Delta \equiv \mu - gb^* - j/g \quad (3.15)$$

Equation (3.11) determines the possible values for K_0 . From these, together with the Gel'fand-Levitan-Marchenko equation [14–16], we may obtain the explicit form of $\phi_{bubble}(x)$

$$\phi_{bubble}(x) = \phi_2 - \frac{K_0^2}{g\omega_0} \frac{1}{\sinh(2K_0s_0)} \{ \tanh[K_0(x + s_0) + \delta] - \tanh[K_0(x - s_0) + \delta] \} \quad (3.16)$$

where

$$K_0s_0 \equiv \tanh^{-1} \left(\frac{m_F - \omega_0}{K_0} \right) \quad (3.17)$$

$$\tanh \delta \equiv \frac{K_0(m_F - \omega_0) - C_0\omega_0}{K_0(m_F - \omega_0) + C_0\omega_0} \quad (3.18)$$

and C_0 is the normalization constant of the bound states. Equation (3.16) may be rewritten as

$$\phi_{bubble}(x) = \phi_2 + \phi_p \{ \tanh(\xi + \xi_0) - \tanh(\xi - \xi_0) \} \quad (3.19)$$

where

$$\xi \equiv K_0x + \delta \quad (3.20)$$

$$\xi_0 \equiv K_0s_0 \quad (3.21)$$

$$\phi_p \equiv -\frac{K_0^2}{g\omega_0 \sinh(2K_0s_0)} \quad (3.22)$$

Looking at (3.19) we notice that, in spite of the presence of fermions, the functional form of the time-independent solution remained unchanged. The effect of the fermions is to rescale the parameters of the “bubble”.

C. Stability of the “bubble” solution

In order to have an asymmetrical potential with a false vacuum located at positive ϕ_2 (see Figure 5), we must adjust the parameters such that [14]

$$2g\bar{\phi}_e(\bar{\phi}_e + a)(\bar{\phi}_e - a) = \mu - gb^* - j/g = \Delta < 0 \quad (3.23)$$

where $\bar{\phi} \equiv \phi + \mu/g$, $a \equiv (\bar{\phi}_0^2 - 1/2)^{1/2}$ and $\bar{\phi}_e$ is an extremum of $V(\bar{\phi})$.

We are now able to obtain, numerically, the solutions of the gap equation (3.11) and the form of the energy of the “bubble” as a function of its radius. Assuming $N = 2$, we can plot these results for the interesting cases $\Delta n = -2, -1, 0$ (see Figures 6 and 7).

Looking at these figures, we immediately see that our expectations are fulfilled in several cases. The reader will have noticed that the existence of a metastable “bubble” is associated with “doping”: for $\Delta n = -2$ (“ground state”) we find just the “sphaleron”.

If we remember that we have an effective theory for the bosons, it should be clear that we are in the same situation as at the end of section 2.B. The differences are in the rescaling of the parameters of $\phi_{bubble}(x)$ and in the form of $E = E[s]$. This latter difference will certainly imply a difference between the lifetimes of the “sphaleron” (the “bubble” of critical radius) and that of the metastable “bubble”. The decay rate may be calculated in the same manner (see equation (2.12)) with E_{sph} replaced with $E_{sph} - E_{mb}$, E_{mb} being the energy of the metastable “bubble”, whenever appropriate.

We should remark that, for temperatures which are low compared with E_{sph}/k_b , the metastable “bubbles” may have an appreciable lifetime. Thus, if we consider small oscillations of these structures around the metastable minimum of $E = E[s]$, ie, oscillations in the size of the “bubble”, we may be able to detect their presence through their charges ³.

IV. FINITE TEMPERATURE AND CHEMICAL POTENTIAL EFFECTS

A natural way to control the “doping” mentioned in the last section consists in the introduction of a chemical potential, ϵ_F . This may be implemented by adding, to our original Lagrangian, a term like [17]

$$\mathcal{L}_{ch.pot.} = \epsilon_F \bar{\psi} \gamma_0 \psi \tag{4.1}$$

³Natural systems for this kind of experiment are doped linear polymer chains [12].

The net effect of this term is to alter the form of our Floquet indices to

$$\alpha_i = (\omega_i - \epsilon_F) T \quad (4.2)$$

We may now study the case of finite temperature by doing the analytic continuation $iT \rightarrow \beta$, so that

$$\alpha_i = -i\beta(\omega_i - \epsilon_F) \quad (4.3)$$

Using the results of *Dashen et al.* [13] for the generating functional of the fermions

$$\mathcal{Z} = \left[\prod_{i=-\infty}^{\infty} (1 + e^{-i\alpha_i}) \right]^N e^{N \sum_{i=-\infty}^{\infty} (i\alpha_i/2)} \quad (4.4)$$

we may calculate the free energy $F = -\frac{1}{\beta} \ln \mathcal{Z}$

$$\begin{aligned} F = & -N(\omega_0 - m_F) - 2N \int_0^\Lambda \frac{dk}{2\pi} (\omega - \epsilon_F) - \\ & - \frac{1}{\beta} \ln \left[\sum_{\{n_{\pm 1}\}} C(\{n_i\}, N) e^{-\beta(n_{+1} + n_{-1})(\omega_0 - \epsilon_F)} \right] - \\ & - \frac{4N}{\beta} \int_0^\Lambda \frac{dk}{2\pi} \ln(1 + e^{-\beta(\omega - \epsilon_F)}) \end{aligned} \quad (4.5)$$

and, finally, the energy due to the fermions, $E_F = \beta \frac{\partial F}{\partial \beta} + F$,

$$E_F = (\langle n_+ \rangle - \langle n_- \rangle)(\omega_0 - m_F) + \frac{NI_2}{\pi} - 4N \left(\frac{\partial \tilde{\Delta}}{\partial \beta} \right) \quad (4.6)$$

where we have introduced the following definitions in expression (4.6)

(a) 1st term: contribution from the discrete levels with

$$\langle n_i \rangle = \frac{\sum_{n_i} C(\{n_i\}, N) n_i e^{-\beta n_i(\omega_0 - \epsilon_F)}}{\sum_{n_i} C(\{n_i\}, N) e^{-\beta n_i(\omega_0 - \epsilon_F)}} \quad (4.7)$$

(b) 2nd term: contribution from the “Dirac sea”.

(c) 3rd term: contribution from the continuum states with

$$\tilde{\Delta} \equiv 2\beta \int_{m_F}^\Omega \frac{d\omega}{e^{\beta(\omega - \epsilon_F)} + 1} \tan^{-1} \left(\frac{K_0}{\sqrt{\omega^2 - m_F^2}} \right) \quad (4.8)$$

where Ω is the cutoff value of the energy (associated to the bandwidth in polymer models).

The energy E_F may be rewritten in terms of the parameter θ_0 as follows

$$E_F = m_F \left\{ (\langle n_+ \rangle - \langle n_- \rangle)(\cos \theta_0 - 1) + \frac{2N}{\pi} \left[\left(\frac{\pi}{2} - \theta_0 \right) \cos \theta_0 + \left(1 + \frac{\pi}{Ng^2} - \gamma \right) \sin \theta_0 - \frac{\pi}{2} \right] \right\} - \frac{4N}{m_F} \frac{\partial \tilde{\Delta}}{\partial \beta} \quad (4.9)$$

where we have introduced the parameter

$$\gamma \equiv \frac{\pi}{Ng^2} \left(\frac{\mu - gb^* - j/g}{\mu + g\phi_2} \right) = \frac{\pi}{Ng^2} \left(\frac{\Delta}{m_F} \right) \quad (4.10)$$

which defines the form of the potential. Adding to (4.9) the bosonic contribution for the energy of the “bubble” we obtain, as our total energy,

$$E = m_F \left\{ \frac{8}{3} \left(\frac{m_F}{g} \right)^2 \sin^3 \theta_0 + 2 \left(\frac{N}{\pi} - \frac{\Delta}{m_F} \right) \sin \theta_0 + N \left(1 - \frac{2\theta_0}{\pi} + \frac{\langle \Delta n \rangle}{N} \right) \cos \theta_0 - \left(\frac{\Delta}{m_F} \right) \ln \left(\frac{1 - \sin \theta_0}{1 + \sin \theta_0} \right) - (N + \langle \Delta n \rangle) - \frac{4N}{m_F} \frac{\partial \tilde{\Delta}}{\partial \beta} \right\} \quad (4.11)$$

where we have defined $\langle \Delta n \rangle \equiv \langle n_+ \rangle - \langle n_- \rangle$, and the gap equation

$$\theta_0 + \gamma \tan \theta_0 + \frac{2\pi}{N} \left(\frac{m_F}{g} \right)^2 \sin 2\theta_0 = \frac{\pi}{2} \left[1 + \frac{\langle \Delta n \rangle}{N} + \frac{4}{m_F} \frac{\partial}{\partial \theta_0} \frac{\partial}{\partial \beta} \tilde{\Delta} \right] \quad (4.12)$$

to be compared with (3.11). We are now ready to analyze the form of $E = E[s]$ in the limits of high and low temperatures.

In the high temperature limit, $\beta|\Omega - \epsilon_F| \ll 1$. Thus, we can approximate the values of $\tilde{\Delta}$ and $\langle \Delta n \rangle$ by

$$\tilde{\Delta} \approx \beta \int_{m_F}^{\Omega} d\omega \tan^{-1} \left(\frac{K_0}{\sqrt{\omega^2 - m_F^2}} \right) = \beta \frac{I_2}{2} \quad (4.13)$$

$$\langle \Delta n \rangle \approx -\frac{\beta(\omega_0 - \epsilon_F)}{1 - \beta(\omega_0 - \epsilon_F)} \quad (4.14)$$

in order to obtain, for the total energy,

$$\begin{aligned}
E = m_F \Bigg\{ & \frac{8}{3} \left(\frac{m_F}{g} \right)^2 \sin^3 \theta_0 + 2 \left(\frac{N}{\pi} - \frac{\Delta}{m_F} \right) \sin \theta_0 + N \left(1 - \frac{2\theta_0}{\pi} - \right. \\
& - \frac{\beta}{N} \frac{(\omega_0 - \epsilon_F)}{1 - \beta(\omega_0 - \epsilon_F)} \Bigg) \cos \theta_0 - \left(\frac{\Delta}{m_F} \right) \ln \left(\frac{1 - \sin \theta_0}{1 + \sin \theta_0} \right) - \\
& - \left(N - \frac{\beta}{N} \frac{(\omega_0 - \epsilon_F)}{1 - \beta(\omega_0 - \epsilon_F)} \right) - 4N \left[\left(\frac{\pi}{2} - \theta_0 \right) \cos \theta_0 + \right. \\
& \left. \left. + \left(1 + \frac{\pi}{Ng^2} - \gamma \right) \sin \theta_0 - \frac{\pi}{2} \right] \right\} \tag{4.15}
\end{aligned}$$

The form of $E = E[s]$ is shown, for various values of the chemical potential, in Figure 8. From this figure, we clearly see that high temperatures prevent the appearance of metastable “bubbles”, just yielding dissociation curves.

For low temperatures, we have $\beta|m_F - \epsilon_F| \gg 1$. The curves for $E = E[s]$ are shown in Figure 9 for the same values of ϵ_F^4 . In this case, temperature plays a small role, as kT is small compared to the other energy scales. Our results are, thus, analogous to the ones we encountered when we considered “doping” via a choice of occupation numbers.

V. CONCLUSION

The aim of this work was the study of metastable systems, with particular interest in the stability of “bubble” structures. The novelty, in comparison with other recent papers, is the presence of fermions and the exciting role they play. As we have seen, the presence of fermions creates a qualitatively different scenario for the evolution of the “bubbles”. Depending almost exclusively on “doping” (relative occupation of bound fermionic states), which may be implemented by means of a chemical potential, we may find different regimes for the stability of the “bubbles”. More precisely, for non-trivial “doping” we encounter a phenomenon of “quantum stabilization” that brings about metastable “bubbles”. This

⁴For $\epsilon_F < 0$, we have the same results with anti-particles in the place of particles.

striking feature, that comes as a consequence of the introduction of fermions, may cause important changes in the physics of the models used to describe the systems mentioned in the Introduction.

These results are a great stimulus for generalizing this work to higher dimensions. However, there are some points in our scheme that require improvement: the inclusion of spinodal decomposition to compete with the nucleation mechanism, and a careful study of tunneling, as a competitor of thermal activation, together with an improvement of some approximations, such as the use of the inverted oscillator [8,9,18], are some of the points to be refined in order to bring the formalism closer to reality.

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Appendix

(a) Definition of the kernels:

$$K_1(x-y) \equiv \int \frac{dk}{2\pi} e^{-ik(x-y)} \left[\frac{\omega_k \cosh(\beta\hbar\omega_k)}{\hbar \sinh(\beta\hbar\omega_k)} \right] \quad (5.1)$$

$$K_2(x-y) \equiv \int \frac{dx}{2\pi} e^{-ik(x-y)} \left[\frac{\omega_k}{\hbar \sinh(\beta\hbar\omega_k)} \right] \quad (5.2)$$

$$(K_i)_{ll'} \equiv \int dx dy \frac{dk}{2\pi} e^{-ik(x-y)} K_i(k) f_l(x - x_{c.m.}) f_{l'}(y - x_{c.m.}) \quad (5.3)$$

$$\tilde{K}_{ll'} \equiv (K_1 - K_2)_{ll'} ; l, l' \neq -1 \quad (5.4)$$

$$Q_l = (K_1 - K_2)_{-1,l} ; l \neq -1 \quad (5.5)$$

where $\{f_l(x)\}$ are the eigenfunctions of the fluctuation operator.

(b) Definition of the inverse scattering integrals [14]:

$$I_0 \equiv \frac{1}{4\pi i} \mathcal{P} \int_{-\infty}^{\infty} dq \left[\frac{P_+(q)}{im_F + q} + \frac{P_-(q)}{im_F - q} \right] + \ln \left(\frac{m_F - K_0}{m_F + K_0} \right) \quad (5.6)$$

$$I_1 \equiv -\frac{1}{2} \left\{ \frac{1}{2\pi} \mathcal{P} \int dq P_+(q) + 2K_0^+ + [(+) \leftrightarrow (-)] \right\} \quad (5.7)$$

$$I_2 \equiv \frac{1}{2} \left\{ \int_0^\Lambda \frac{k dk}{\sqrt{k^2 + m_F^2}} \left(\mathcal{P} \int_{-\infty}^{\infty} dq \frac{P_+(q)}{k - q} + 2\arctan \left(\frac{K_0^+}{k} \right) + [(+) \leftrightarrow (-)] \right) \right\} \quad (5.8)$$

$$I_3 \equiv -\frac{1}{2} \left\{ \frac{2}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dq q^2 P_+(q) - \frac{8}{3} (K_0^+)^3 + [(+) \leftrightarrow (-)] \right\} \quad (5.9)$$

where \mathcal{P} means principal value, $K_0^+ = K_0^-$ and $P_\pm(q) \equiv \ln(1 - |r_\pm(q)|^2)$.

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Figure Captions:

Figure 1: Form of the potential.

Figure 2: The “sphaleron” solution.

Figure 3: “Bubble” energy as a function of its radius.

Figure 4: Decay rate as a function of time.

Figure 5: Form of $[V(\phi) - V(\phi_2)]$.

Figure 6: Numerical solution of (3.11) for (a) $\Delta n = 0$, (b) $\Delta n = -1$ and (c) $\Delta n = -2$.

Figure 7: Energy of the “bubble” as a function of its radius for (a) $\Delta n = 0$, (b) $\Delta n = -1$ and (c) $\Delta n = -2$. Equation (3.17) imposes the restriction $s_0 \geq 1/2m_F$.

Figure 8: Energy of the “bubble” as a function of its radius in the limit of high temperature. (a) $m_F < \epsilon_F < \Omega$. (b) $\omega_0 < \epsilon_F < m_F$. (c) $0 < \epsilon_F < \omega_0$.

Figure 9: Energy of the “bubble” as a function of its radius in the limit of low temperature. (a) $m_F < \epsilon_F < \Omega$. (b) $\omega_0 < \epsilon_F < m_F$. (c) $0 < \epsilon_F < \omega_0$.

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